Bottle experiment of Clément and Desormes with SensorTag.

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The heat capacity ratio of air (Laplace's coefficient) $\gamma = c_p/c_v$ is an important value in thermodynamics. It enters, for example, in the atmosphere properties and in the speed of sound. There is a lot of literature on the student lab measurements of this value, one modification of the original experiment of Clément and Desormes being accessible now nearly for everybody[1,2]. Today one can use the pressure sensor of a smartphone or other available device (here SensorTag CC2650STK of Texas Instruments) and phyphox software[3,4] to control the experiment.

THEORETICAL BACKGROUND

The idea of the experiment is to rapidly (adiabatically) compress a plastic bottle and measure the evolution of the pressure inside it. Initially air in the bottle is at the atmospheric pressure p_0 and the room temperature T_0. The adiabatic compression (A–B in Fig. 1) from p_0 to p_1 produces heating of the air inside the bottle from T_0 to T_1. According to the Laplace's law $p^{1-\gamma}T^{\gamma} = const$ for the adiabatic process of ideal gases

$$\left(\frac{p_1}{p_0}\right)^{\gamma-1} = \left(\frac{T_1}{T_0}\right)^{\gamma} . \tag{1}$$

During the following cooling back to the room temperature T_0 at the constant volume with the corresponding reduction of the pressure to p_2 (B–C in Fig. 1) air in the bottle serves as *a thermometer of itself* (according to Gay-Lussac's law its pressure p is proportional to the temperature T):

$$\frac{p_1}{p_2} = \frac{T_1}{T_0} . \tag{2}$$

So, the first adiabatic step A–B of the experiment creates heating to T_1 depending from γ and the second step B–C measures this temperature. From two equations (1) and (2) one

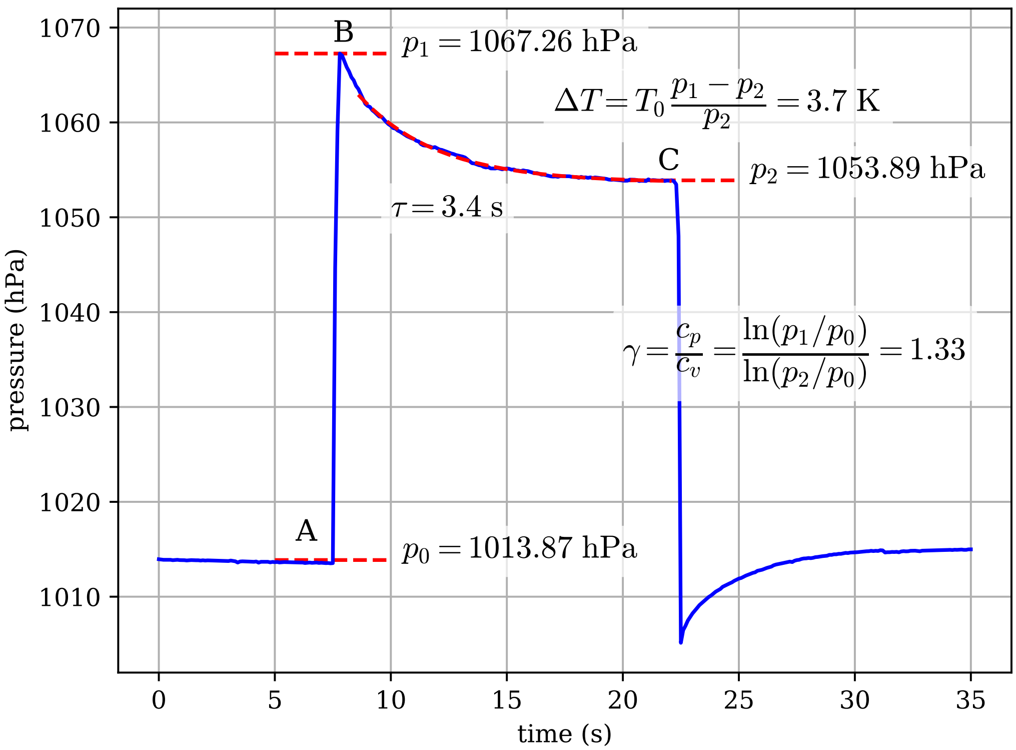

FIG. 1. Experiment of Clément and Desormes using a plastic bottle. A–B: The adiabatic compression of the bottle; B–C: Cooling down to the room temperature at the constant volume.

can easily get

$$\gamma = \frac{\ln(p_1/p_0)}{\ln(p_2/p_0)} \ . \tag{3}$$

The derivation of (3) by Mottmann[1] is a little different. He used $pV^\gamma = const$ and a virtual relation between states A and C. In this way one do not see well enough what really happens and what is the meanings of the experimental step B–C.

One gets heating involved into the experiment from equation (2)

$$\Delta T = T_1 - T_0 = T_0 \frac{p_1 - p_2}{p_2} \ . \tag{4}$$

EXPERIMENT

In order to measure pressure with a smartphone or a SensorTag the device should be placed into a hermetically closed volume connected to the bottle. In our case the SensorTag put into a small jar, its cap being connected to the bottle cap by a tube (see Fig. 2).

FIG. 2. Realization of the experiment: The SensorTag (orange) is visible inside the small jar (at the left). Its cap is connected to the bottle (at the right). The ruler is served to apply pressure to the bottle.

Pressure is applied to the bottle by a ruler. One may not touch the bottle by hand which would heat the air inside it and falsify the measurements. The bottle deformation should be held constant during the temperature relaxation B–C. A program for a smartphone or a tablet controlling the experiment under the phyphox app was created not only to measure the time evolution of pressure, but also to extract γ and ΔT (see Fig. 3) using equations (3) and (4). The program code together with a description how to use it can be found in the phyphox forum.[4] Phyphox allows to save the measured $p(t)$, which can be analyzed using Python (Fig. 1). Experimental stage B–C is well fitted by an exponential with characteristic time $\tau = 3.4$ s.

The experimental value $\gamma = 1.33$ is closed to the theoretical 1.4 for diatomic gases. As to $\Delta T \sim 3.7°$C, it is worse to measure. Such a measurement should use a very light temperature sensor capable to follow the fast (some seconds) evolution of the air temperature. It might

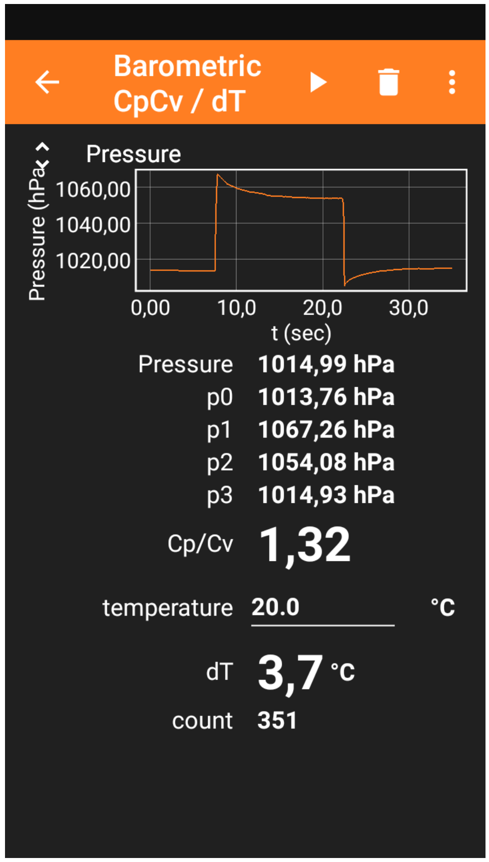

FIG. 3. Smartphone screen of the phyphox program measuring $p(t)$ and extracting the key pressures p_0 to p_3, $\gamma = c_p/c_v$ and ΔT.

be a project for a future work.

[1] J. Mottmann, "Laboratory experiment for the ratio of specific heats of air," Am. J. Phys. **63**, 259–260 (1995).

[2] Michael J. Moloney and Albert P. McGarvey, "A Simplified Adiabatic Compression Apparatus," Phys. Teach. **45**, 452–453 (2007).

[3] Christoph Stampfer, Heidrun Heinke and Sebastian Staacks, "A lab in the pocket," Nat. Rev. Mater. **5**, 169–170 (2020).

[4] `phyphox` (physical phone experiments) Web Site, <https://phyphox.org/>.